



PAPER

Localization of edge state in acoustic topological insulators by curvature of space

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E-mail: hwwu@aust.edu.cn**Keywords:** localized edge state, acoustic topological insulators, curved spaceSupplementary material for this article is available [online](#)

Abstract

Topological insulators (TIs) with robust boundary states against perturbations and disorders have boosted intense research in classical systems. In general, two-dimensional (2D) TIs are designed on a flat surface with special boundary to manipulate the wave propagation. In this work, we design a 2D curved acoustic TI by perforation on a curved rigid plate to localize the edge state by means of the geometric potential effect, which provide a unique approach for manipulating waves. We experimentally demonstrate that the topological edge state in the bulk gap is modulated by the curvature of space into a localized mode, and the corresponding pressure distributions are confined at the position with the maximal curvature. Moreover, we experimentally verify the localized edge state is still topologically protected by introducing defects near the localized position. To understand the underlying mechanism for the localization of the topological edge state, a tight-binding model considering the geometric potential effect is proposed. The interaction between the geometrical curvature and topology in the system provides a novel scheme for manipulating and trapping wave propagation along the boundary of curved TIs, thereby offering potential applications in flexible devices.

1. Introduction

Topology, which is a mathematical concept for describing the space-preserved property in continuous deformation, has been introduced into condensed-matter physics to investigate various phenomena, including the quantum integer Hall effect [1, 2], quantum spin Hall effect [3, 4], and topological insulators (TIs) [5, 6]. In the past several decades, TIs have constituted an expanding research field in condensed matter, and the robust transport effect of boundary states against disorders has attracted intense interest in classical systems [7–12]. Inspired by these concepts, analogous topological optics/acoustic are becoming a hot notion throughout physics in a variety of frontier domains [13–29], such as one-way propagation [13, 14], communications [15, 16], and acoustic-noise reduction [17] and so on. Similar to the electrons propagating in a crystal, sound in the phononic crystal will also experience a periodic potential [18–20], and the physical performance can be delineated by the energy band structure, such as the concept of topology [21–23]. Topology derives from the winding of eigenmodes in momentum space and usually be characterized by the topological invariant, and each band has a topological invariant [24]. Usually, if there are two bands can be adiabatically transformed each other, it can be regard as topologically equivalent [25], and will appear band gap remains open by connected two equivalent TIs [26]. Typically, the gapless boundary modes is protected by the topology, and the modes in the boundary possess robust characteristics which are insensitive to structural disturbance [27–29].

As another interesting topic in wave propagation, curved space has been studied extensively for defining the transported properties of quantum particles [30–32] and classical waves [33–45] that are confined on a

curved surface. As early as 1981, pioneering work presented by da Costa [32] indicated that the motion of a quantum particle constrained on a curved surface by a physical confining potential experiences effective geometric potential. With reference to the similarity between the Schrödinger equation and paraxial Helmholtz equation, optical analogues of the quantum geometric potential have been studied theoretically and experimentally in optical systems for light wave packets constrained on thin dielectric guiding layers [34–40]. With the aid of the geometric potential effect, numerous quantum phenomena have been migrated to classical systems, such as optical Bloch oscillation [33, 34], Zener tunneling [37, 38], and the Wolf effect [40]. In recent years, a curved surface plasmon polariton (SPP) system was proposed to investigate the geometric potential effect [42–44]. Unlike photons that are squeezed on a curved surface by a thin dielectric layer, SPPs are intrinsically two-dimensional (2D) quasi-particles without any squeezed potential requirements. Based on this versatile platform, the geometric potential effect has been experimentally demonstrated to manipulate SPP propagation along a curved interface between metal and dielectric materials. Furthermore, with the research upsurge in TIs, geometric potentials have been used to explore topological phases in curved-space systems. For example, by setting a cylindrical thin waveguide layer with an imprinted curved Su–Schrieffer–Heeger (SSH) binary lattice, Lustig *et al* [45] demonstrated that the space curvature can induce topological edge states, topological phase transitions, Thouless pumping, and localization effects. The interaction between TI and space curvature provide more opportunities for manipulating the wave propagation in the TI [46–48].

In this work, we designed a curved 2D TI in an acoustic system by perforating the curved surface of a rigid wall based on the lattice distribution of the 2D SSH model. We experimentally observed that a topological edge state with the highest eigenfrequency is distinct from the edge states and exhibits strength mode localization induced by the geometric curvature. By introducing near and far defects into the structure, we also demonstrated that the localized topological edge state is still robust against disturbances. Furthermore, the tight-binding model considering the geometric potential effect was used to reveal the physical mechanism underlying the localized edge state. The results indicate that the nonhomogeneous onsite energy leads to mode localization. Although our study was carried out in the context of acoustics, the concepts involved are universal, with manifestations in many branches of physics, including electronic, electromagnetic and elastic systems. Curved TIs provide the unique ability to manipulate and trap the waves propagated along the edges of TIs, and offer potential applications in flexible devices.

2. Construction of curved 2D TI

Our 2D curved TI was designed by perforating a curved surface of a rigid wall to form a square lattice distribution, as illustrated in figure 1(a). The yellow region represents the curved rigid wall with a length of 65.8 cm, width of 48.4 cm, and thickness of 3.5 cm. The shape of the bump could be described by the function $z = H \cos(\pi x/L)$, where the bump height $H = 5$ cm, bump range $-L/2 < x < L/2$, and $L = 40$ cm. The depth and radius of the holes were $h = 3$ cm and $r = 0.3$ cm, respectively, as indicated in the right inset. The lattice constant was $a = 4$ cm. The distance between the neighboring holes in a unit cell was represented as d , and the topological phases could be changed by tuning the ratio d/a . It should be explained why the 2D curved TI was designed using the perforated method. The reason is that the holey structure could support the spoof acoustic surface waves confined on the curved surface to satisfy the condition of generating the geometric potential effect. Such holey structures are common in acoustic metamaterials. Previous work based on this type of holey structure on a 2D flat surface realized acoustic topological corner states [26]. In our work, the entire rigid wall included two types of hole arrays ($d/a = 0.75$ and 0.25) to form an interface, as indicated by the bright trace in figure 1(a). The two insets on the right of figure 1(a) depict the air domain of the unit cell in the cyan region.

Prior to investigating the topological properties of 2D curved TIs, we recall the band structure of the square lattice distributed on a flat surface corresponding to $H = 0$ cm, and demonstrate the band inversion and topological phase transition when changing the ratio d/a in figure 1(b). The red pentagrams show the band diagram of the square lattice for the ratio $d/a = 0.5$. The first Brillouin zone is displayed in the inset of figure 1(b). The blue dashed lines represent the dispersion relationships for the sound propagating in the air background. When comparing the band structure and air line, the feature of spoof acoustic surface waves could be observed [26]. Furthermore, the bands s and p_x were degenerated along the $X-M$ direction for the ratio $d/a = 0.5$. When the distance between the four holes in the unit cell was decreased to $d/a = 0.25$ or increased to $d/a = 0.75$, the degenerate band was separated to form a bandgap, as indicated by the black dots. To demonstrate the topological phase transition from trivial to nontrivial by increasing the distance from $d/a = 0.25$ – 0.75 , we first present the parities of the pressure fields in the unit cell at point X , as illustrated in figure 1(c), for calculating the 2D Zak phases. It is obvious that the eigenstate with a high frequency possessed odd parity corresponding to the p_x band, whereas the eigenstate with a low frequency had even

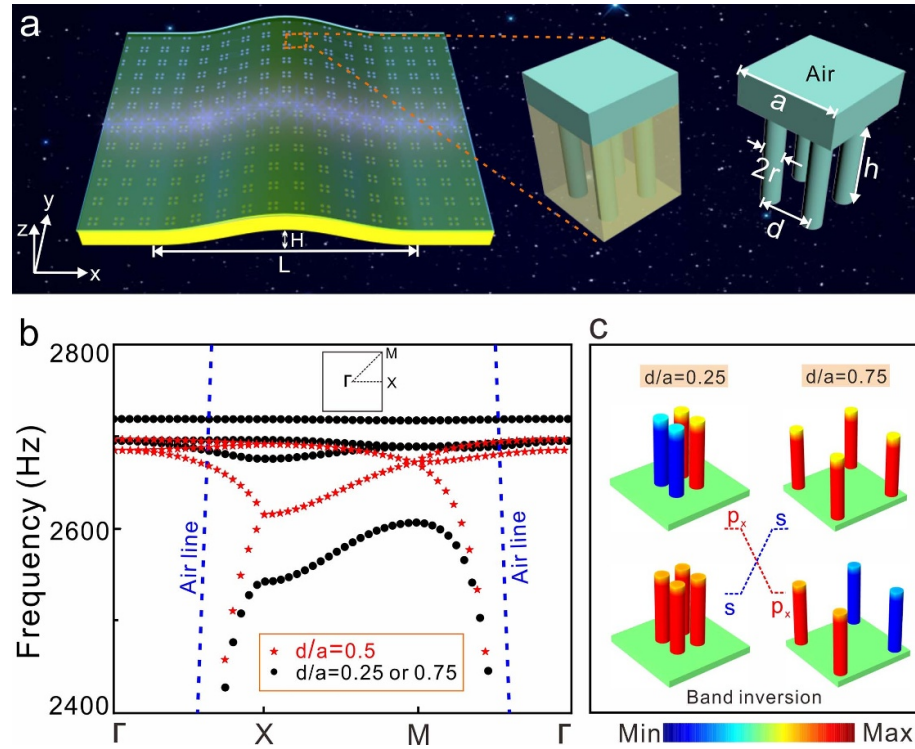


Figure 1. Unit cell of 2D curved acoustic TI and band inversion. (a) Schematic of curved TI by perforation on curved rigid plate with height H and span L . The right insets show the details of the unit cell as the lattice length a , hole radius r , hole depth h , and hole distance d . The yellow and cyan areas represent the rigid wall and air, respectively. The interface between the topological trivial and nontrivial lattices is indicated by the light trace. (b) Band structure of square lattice for different ratios $d/a = 0.5$ (red pentagrams) and $0.25/0.75$ (black dots). The blue dashed lines represent air lines and the inset shows the first Brillouin zone. (c) Band inversion between topological trivial phase ($d/a = 0.25$) and nontrivial phase ($d/a = 0.75$). The colors correspond to the pressure distributions at point X .

parity corresponding to the s band for $d/a = 0.25$. When increasing the distance between the four holes in the unit cell to $d/a = 0.75$, the energy bands were inverted, as can be observed in figure 1(c). To characterize the topological phase transition rigorously, the 2D Zak phases were calculated by the parities at high symmetric points, as follows [26]:

$$P_m = \frac{1}{2} \left(\sum_n q_m^n \bmod 2 \right), \quad (-1)^{q_m^n} = \frac{\eta(X_m)}{\eta(\Gamma)}, \quad (1)$$

where η denotes the parity associated with π rotation at high symmetric points and m represents the x or y direction. The summation $\sum_n q_m^n$ was taken over all bands from the 1st to n th. For the square lattice with C_{4v} symmetry, the Zak phase P_x was identical to P_y in the y direction; that is, $P_x = P_y$. According to equation (1) and all eigenstates at high symmetric points, we could obtain the 2D polarization $\mathbf{P} = (0, 0)$ for the ratio $d/a < 0.5$ and $\mathbf{P} = (1/2, 1/2)$ for the ratio $d/a > 0.5$. The results reveal that the structures with ratios of $d/a < 0.5$ and $d/a > 0.5$ corresponded to topological nontrivial and trivial insulating phases, respectively.

Subsequently, we firstly investigated the influence of the geometrical curvature on the band structure of the 2D curved TI for the square lattices distributed on a curved surface by numerical simulations. As a reference, we constructed a square crystal composed of a $6a \times 16a$ nontrivial region with $d/a = 0.75$ near a same-sized trivial region with $d/a = 0.25$ on the flat holey plate to form an interface, as illustrated in figure 2(a). Figure 2(b) presents the curved case, which had the same lattice distribution as the flat holey plate, with 480 holes located on the bump surface. Figures 2(c) and (d) depict the discrete eigenfrequencies for the flat and curved structures, respectively. In this simulation, we set the boundary conditions around the entire structure as plane wave radiation boundaries to focus only on the edge states along the interface between the topological trivial and nontrivial parts for a clear observation of the influence of the geometrical curvature on the edge states. The difference between the radiation boundary condition and air area around the structure is illustrated in figure S1 of the supplementary material. It can be observed that there are 15 edge states supported by the interface presented in figure 2(c), which are marked by red dots. These edge states are clarified by their eigenstates in figure S2 of the supplementary material. For the curved case, as illustrated in figure 2(d), the number of edge states was not changed, but the eigenfrequencies of the edge

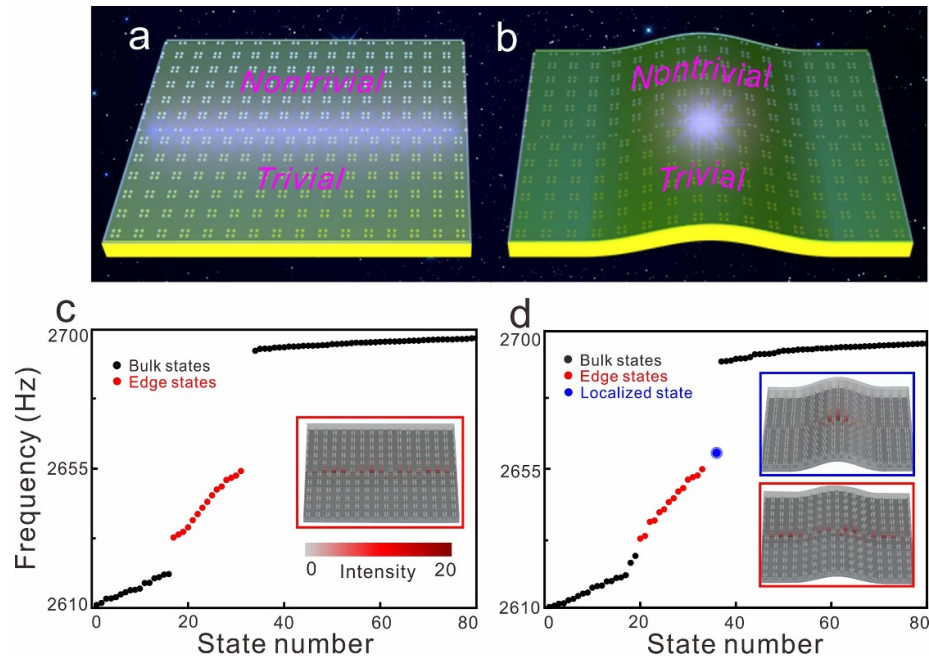
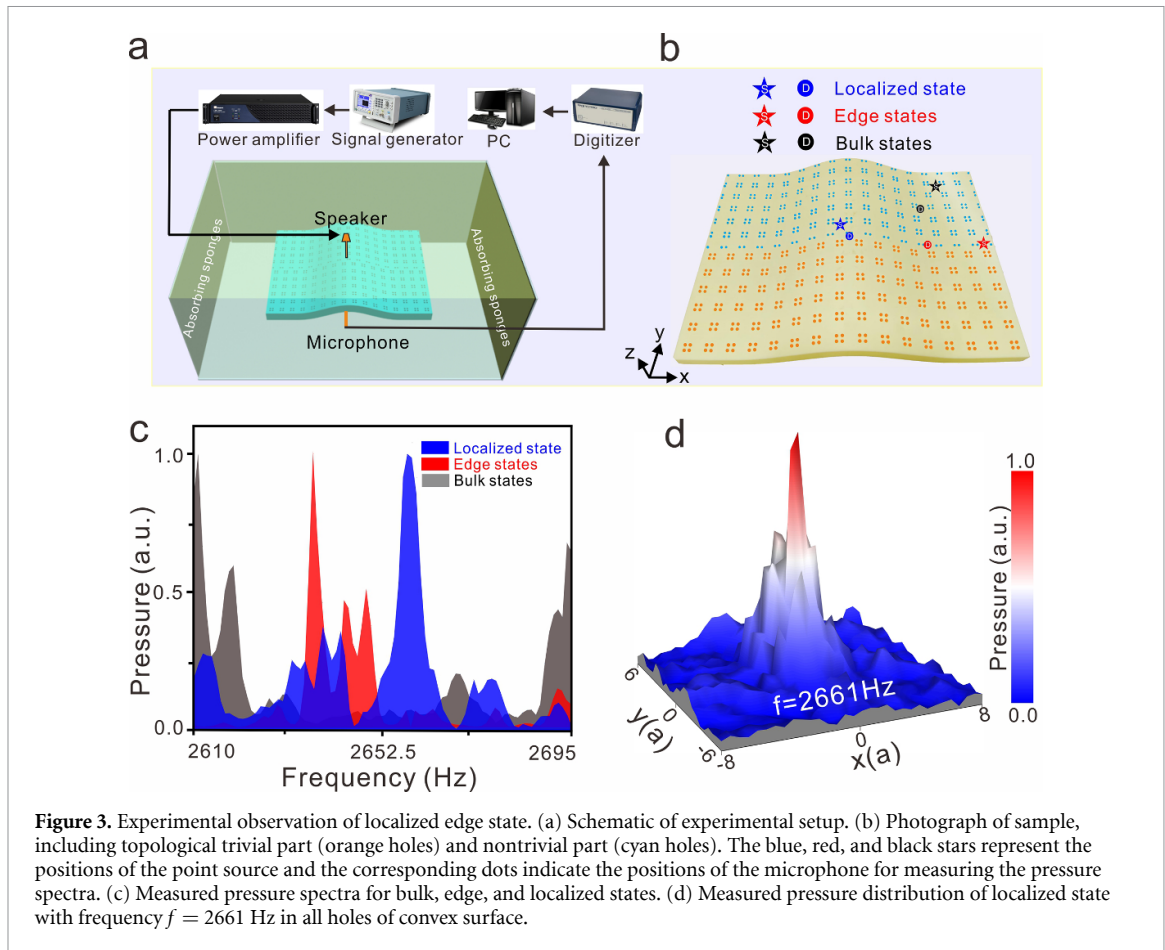


Figure 2. Topological localized state induced by geometrical curvature. Structural diagrams for (a) flat and (b) curved holey TIs. The eigenfrequency distributions for the flat and curved structures are depicted in (c) and (d), respectively. The blue, red, and black dots represent the localized, edge, and bulk states, respectively. The insets with the red boxes in (c) and (d) correspond to the pressure distributions of the 12th edge states of the flat and curved structures, respectively. The inset with the blue box in (d) represent the pressure distribution of the localized edge state.

states were obviously increased. By determining the mode distributions of these eigenstates corresponding to the eigenfrequencies, as indicated in figure S2, an interesting phenomenon was observed whereby a localized state emerged at the top of the bump, as shown in the inset of figure 2(d). The eigenfrequency of this localized state is marked by a blue dot in figure 2(d). The mode distributions of the topological edge states corresponding to the 12th edge states are depicted in the insets of figures 2(c) and (d). Comparing the eigenfrequency distributions in figures 2(c) and (d), the edge states exhibited energy degeneration in the curved case, which is verified by the mode distributions in c2–12 of figure S2 in the supplementary material. The reason is that the bilaterally symmetrical geometrical shape led to bilaterally symmetrical distributions of the onsite energy of the holes on the curved surface; consequently, the eigenstates of the holey structure located at the two slopes underwent energy degeneration. In particular, the eigenstate located at the top of the bump with the maximal onsite energy was isolated from the other degenerated edge states to form the localized state. Furthermore, the evolution of the eigenfrequency when increasing the height H of the curved structure is illustrated in figure S3 in the supplementary material. Furthermore, we also calculate the polarization P_x and P_y for the holes with different onsite energies along x direction to check the topological property. As can be seen from figure S4 of the supplementary material, the polarization $P_y = 0.5$ along y direction keep unchanged for introducing varied geometrical potential energies to holes along x direction, while the polarization P_x decreases from 0.5 with increasing the additional potential energies. The unchanged P_y indicates that the edge states always present for bending structure along x direction due to the unchanged TI structure in y direction. Furthermore, the varying polarization P_x indicates that the localized edge state comes from the distraction of geometrical potential, which breaks the topological property. Thus, it needs to be noticed that the localized edge state is different with conventional topological corner state ($P_x = P_y = 0.5$). The influence of geometrical curvatures on the band structure and mode distribution with tight-binding model is presented in the following theoretical section to understand the localization of edge state.

3. Observation of localized edge state

Next, we experimentally measured the topological localized state in a curved 2D TI. The holey structure was constructed via three-dimensional (3D) printing of a curved epoxy resin plate and perforation on the curved plate surface. Epoxy resin can be treated as a rigid wall compared to air. Figure 3(a) presents the measurement setup. The sample was surrounded by absorbing sponges and a point source was placed at a distance of 3 cm from the curved surface of the sample. A microphone was placed at the back of the sample to detect the pressure amplitudes. Figure 3(b) indicates the placements of the source and microphone,



marked by colored pentagrams and circles, respectively, for measuring the pressure amplitudes of the bulk, edge, and localized states. The transmission spectra of these eigenstates are depicted in figure 3(c). It can be observed that the transmission peaks of the edge and localized states were clearly located in the bandgap of the bulk band. In particular, the frequency $f = 2661$ Hz of the localized edge state agreed very well with the simulation frequency $f = 2660.7$ Hz. To observe the localized edge state in real space clearly, we also measured the pressure distributions of all holes on the curved surface at $f = 2661$ Hz in figure 3(d). The result demonstrated the field confinement of the localized state at the structure top with the maximal curvature. Furthermore, to compare the measured and simulated results, we scanned the pressured field amplitude profile in each hole along two directions, as indicated by the cyan and orange dashed lines marked with b and c in figure 4(a). Figures 4(b) and (c) present the normalized pressure field distributions along the b and c directions, where the black solid lines represent the distribution in the simulation and the red dotted dashed lines correspond to the measured results. The measured results indicate that the pressure fields were mainly localized at the holes of the top, which were consistent with the simulated results.

To demonstrate the localized edge state is robust against defects, we introduced near and far defects into the curved structure by inserting solid cylinders into the holes, as indicated by the cyan and orange dashed circles in figure 4(d). The bottom-right inset clearly shows the positions of the two types of defects. Figure 4(e) displays the transmission spectra of the localized state with and without the near defects. The black solid line is the transmission spectrum of the perfectly curved structure and the transmission peak was located at $f = 2661$ Hz. When introducing the near defects into the curved structure, the magnitude of the peak was impregnable, as indicated by the blue dashed line in the simulation and red dashed line in the experiment. Figure 4(f) presents a similar phenomenon for the introduction of the far defects. These results suggest that the uplifted edge state can be localized and is robust against near and far defects.

4. Tight-binding model considering geometric potential effect

To understand the physical mechanism of the topological localized edge state induced by the curved space, we theoretically calculated the eigenfrequencies of the edge states based on a tight-binding model by considering the geometric potential of the curved space. For this purpose, we first performed conformal

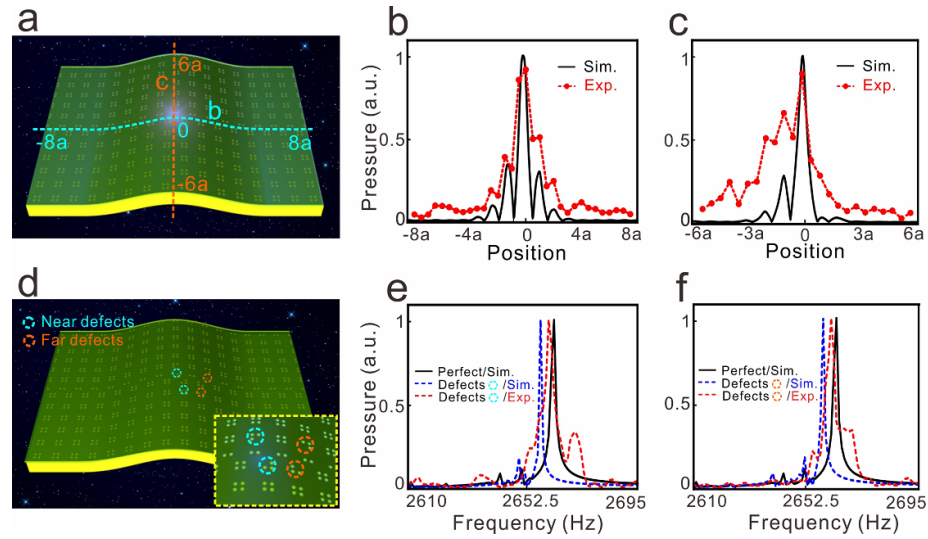


Figure 4. Robustness of topological localized edge state against defects. Measurements of the pressure distributions along the cyan and orange dashed lines b and c in (a). The simulated (black solid line) and experimentally measured (red dot-dashed line) results along b and c are shown in (b) and (c), respectively. (d) Schematic for introducing near and far defects into curved TI, indicated by cyan and orange circles. The inset shows the amplification of the structural top. Transmission spectra for the structure including (e) near defects and (f) far defects obtained by simulation (blue dashed line) and experiment (red dashed lines). The black solid lines represent the results of the perfect structure as a reference.

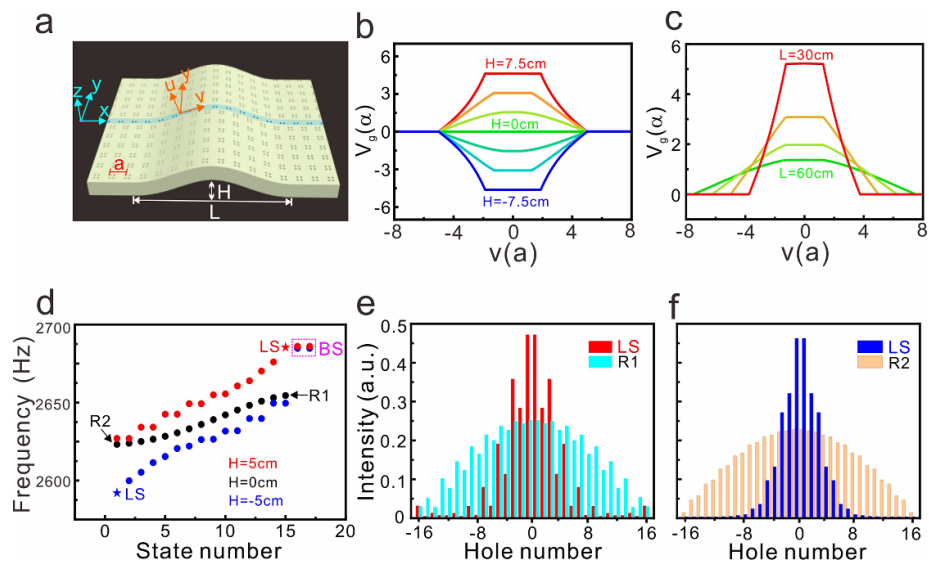


Figure 5. Theoretically calculated geometric potentials and eigenstate distributions of curved structure. (a) Geometry and coordinate system of 2D curved structure, where (x, y, z) and (v, y, u) represent the Cartesian and curvilinear coordinates, respectively, H and L correspond to the height and span of the bump, respectively, and a is the lattice constant. Geometric potential as function of curvilinear coordinate v for different (b) heights and (c) spans. (d) Eigenfrequencies of 1D hole array (as marked by cyan strip in (a)) for different heights $H = 5$ cm (red dots), 0 cm (black dots), and -5 cm (blue dots) based on tight-binding model. The top right dashed box depicts the boundary states, and the red and blue pentagrams marked with 'LS' indicate the topological localized states for heights $H = 5$ cm and -5 cm, respectively. 'R1' and 'R2' indicate the eigenstates with the maximal and minimal frequencies of the flat structure as references. Comparison of mode distributions of localized state marked by red 'LS' and reference 'R1' in (e), and localized state marked by blue 'LS' and reference 'R2' in (f).

transformations between the Cartesian coordinates (x, y, z) and curvilinear coordinates (v, y, u) indicated in figure 5(a) using the relational expression $z + ix = R(v) \exp[(u + iv)/R(v)]$, where $R(v)$ represents the radius of the curvature varied with the curvilinear coordinate v . According to the conformal transformation, and considering that the spoof acoustic surface waves only propagated along the interface between the topological trivial and nontrivial parts owing to the confinement of the bulk bandgap, the spoof acoustic surface wave was expressed as $p(v, u) = A(v, u) \exp(i\beta v)$ under paraxial approximation, where $A(v, u)$ is the

slowly varying amplitude and β is the wavevector. By applying the slowly varying envelope approximation to the 2D scalar wave equation, the acoustic surface wave yielded the acoustic Schrödinger equation [37, 44]:

$$i\hbar \frac{\partial A}{\partial v} = -\frac{\hbar^2}{2n_{sw}} \frac{\partial^2 A}{\partial u^2} + \frac{n_{sw}^2 - n^2(v, u)}{2n_{sw}} A - \frac{un^2(v, u)}{R(v)n_{sw}} A, \quad (2)$$

where $\hbar = 1/k_0$ and k_0 is the wavevector of free space. Further details can be found in the supplementary material. In this case, $n(v, u) = k(v, u)/k_0$ corresponds to the refractive index of the holey materials, and $n_{sw} = \beta/k_0$. For the spoof acoustic surface waves confined on the curved surface, the second term on the right of equation (2) corresponds to the confining potential $V_c(v, u) = [n_{sw}^2 - n^2(v, u)]/(2n_{sw})$. The last term is the geometric potential [37] $V_g(v, u) = -un^2(v, u)/R(v)n_{sw}$. The geometric potential is mainly dependent on the curvature $R(v)$ and can be expressed as $V_g(v) = -\alpha/R(v)$ for the spoof surface wave propagated on the curved surface, with $\alpha = un^2(v, u)/n_{sw}$ as a parameter. For simplicity, we set the value as $\alpha = 10$.

For our structure described by the function $z = H \cos(\pi x/L)$, we calculated the geometric potential for different heights H along the curvilinear coordinate v when the span $L = 40$ cm, as illustrated in figure 5(b). It is obvious that the geometric potential was 0 for $H = 0$ cm, corresponding to a flat structure. When increasing the height H to 2.5 cm, the geometric potential gradually increased along the left structural slope, and the maximal value was attained at the top of the bump with the maximal curvature. Thereafter, the geometric potential gradually decreased from the maximal value to 0 along the right slope. When increasing the height H to 7.5 cm, a higher potential barrier was formed along the curvilinear coordinate v . Conversely, when changing the sign of the height H to negative, a potential well was formed for the concave surface, as indicated in figure 5(b). To clarify the influence of the curved span L on the geometric potential, we also calculated the geometric potentials when decreasing the span from $L = 60$ cm to 30 cm with a constant height of $H = 5$ cm, as depicted in figure 5(c). It can be observed that the potential barrier was increased when decreasing the span. For the concave surface, the depth of the potential well decreased when decreasing the span L ; this result is not provided here.

It is well known that topological edge states are well confined in one-dimensional (1D) holes at the interface between the topological nontrivial and trivial parts owing to the bulk–boundary correspondence. For simplicity, we performed 1D tight-binding approximation to calculate the eigenfrequencies of the 1D hole array at the curved interface by introducing discrete geometric potentials into the onsite energy (that is, resonant frequency) of the holes. It should be noted that the resonant frequency of the hole was 2685 Hz, and the coupling coefficients between two holes were 50 Hz for the ratio $d/a = 0.25$ and 15 Hz for $d/a = 0.75$ obtained by the simulation. When the structure exhibited geometric curvature, the onsite energy of the holes could be modified by adding relevant discrete geometric potentials. Coupling coefficients are recognized as invariable for a weak curvature. Figure 5(d) depicts the eigenfrequencies of the 1D hole array for different heights of $H = -5$ cm, 0 cm, and 5 cm when the span was $L = 40$ cm. The black dots represent the bulk states of the 1D flat hole array. Notably, the bulk states in the 1D hole array of the theoretical model corresponded to the edge states of the 2D topological nontrivial hole array, as indicated in figure 5(a) [27]. Comparing the bulk states (black dots) in figure 5(d) and the edge states (red dots) in figure 2(c), it can be observed that the eigenfrequencies of the bulk states in the 1D hole array were consistent with those of the edge states of the 2D hole array. The correspondence between the bulk states of the 1D hole array and edge states of the 2D hole array could also be verified by comparing their mode distributions, as illustrated in c1 to q1 of figures S2 and S4. In particular, the boundary states of the 1D hole array owing to the open boundary conditions are presented in the dashed box of figure 5(d), which are marked as ‘BS’. However, the corresponding corner states in the 2D hole array are not presented in figure 2(c) owing to the use of radiation boundary conditions.

The above discussion has demonstrated that the tight-binding model of a 1D hole array is valid for studying the edge states of a 2D hole array along the interface in a curved space. According to figure 5(d), when the curved height was increased to $H = 5$ cm, the eigenfrequencies of the bulk states were increased universally and the eigenstates with low frequencies resulted in energy degeneration. All of the mode distributions of the eigenstates are depicted in figure S6. However, the boundary states overlapped in the case of $H = 0$ at 2685 Hz (the resonant frequency of the hole). This is because the holes supported the boundary states located at the flat regions at the two ends, the onsite energy of which was not modified by the geometric potential. As illustrated in figure 2(d), the curved holey structure was bilaterally symmetric and the eigenstates supported on the bilateral slopes exhibited energy degeneration. Subsequently, when the bulk state (labeled with the red ‘LS’ in figure 5(d)) with the maximal eigenfrequency was isolated, its mode was confined at the top of the curved structure with the maximal geometric potential, as can be observed in

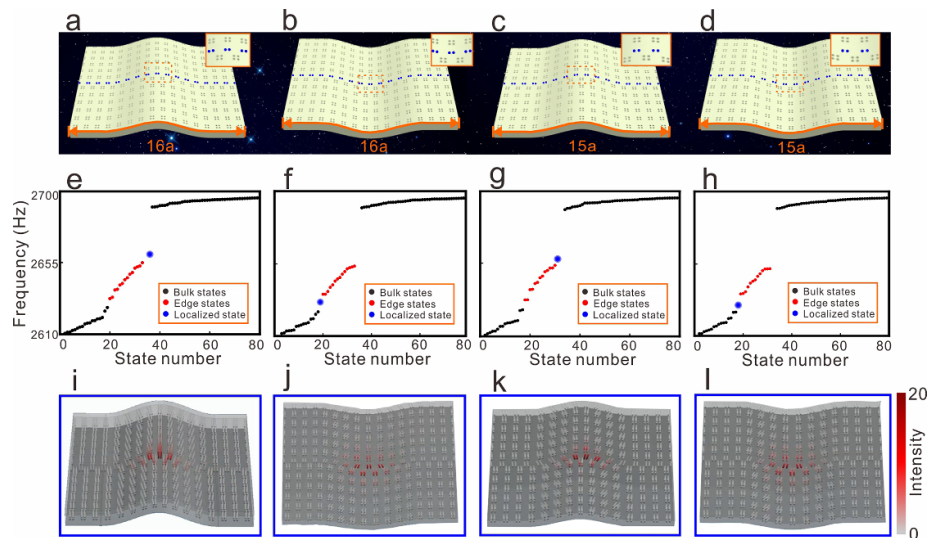


Figure 6. Eigenfrequencies and mode distributions for different curved directions and lattice distributions. (a)–(d) Schematics of 2D curved structures, where (a) and (b) depict 16 unit cells along the curved surface but different curved directions, (c) and (d) show 15 unit cells along the curved surface with different curved directions, and the top-right orange dashed boxes indicate the details of the lattice distribution at the top or bottom of the curved surface. Figures (e)–(h) present the eigenfrequencies of the curved structures in (a)–(d), respectively, and (i)–(l) depict the mode distributions of the topological localized edge states supported in the corresponding curved structures.

figure S6 of the supplementary material. Furthermore, the eigenfrequencies of the concave structure when the curved height was set to $H = -5$ cm are denoted by the blue dots in figure 5(d). It can easily be observed that the eigenfrequencies were decreased when adding a negative geometric potential (as indicated in figure 5(b)) to the resonant frequencies of the holes. Interestingly, the localized state emerged at the bottom of the curved structure with the minimum eigenfrequency, which is marked by the blue ‘LS’ in figure 5(d). Similarly, the eigenstates with higher frequencies exhibited energy degeneration. All of the eigenstates are depicted in figure S7 of the supplementary material. Figure 5(e) presents the intensity distributions of the localized state, marked by the red ‘LS’ in the curved structure ($H = 5$ cm), and the reference state, marked by ‘R1’ in the flat structure. The pressure fields of the topological localized state were effectively confined at the structural top compared to the field distribution of the reference state ‘R1’. We also compared the intensity distributions of the localized state, marked by the blue ‘LS’ in the concave structure ($H = -5$ cm), and the reference state, marked by ‘R2’ in the flat structure, as illustrated in figure 5(f).

The proposed theoretical model suggested that the positions of the holes on the curved surface affected the distribution of the eigenstates, and the curved directions (convex and concave) determined the eigenfrequency of the localized state. To verify the theoretical predictions, we also simulated the curved structures with different curved directions and different hole distributions, as depicted in figure 6. The structures in figures 6(a) and (b) had the same lattice distribution but different curved directions, whereas the structures in figures 6(a) and (c) had the same curved direction but different lattice distributions. In figures 6(a)–(d), 16a and 15a denote 16 cells and 15 cells along the curved surface, respectively. The top right insets display the details of the structural top. Figures 6(e)–(h) present the eigenfrequencies of the 2D curved TI structures. The localized state had the maximum frequency in the bandgap for the convex structure, but the minimum frequency in the bandgap for the concave structure. In general, a convex structure expands the highest edge state to form a topological localized edge state, whereas a concave structure reduces the lowest edge state to form a topological localized state. The results were consistent with our theoretical predictions. Furthermore, comparing the eigenfrequencies of the curved structures in figures 6(e) and (g) (or figures 6(f) and (h)), it can be observed that the eigenfrequencies exhibited a small frequency shift owing to the location shift of the holes on the curved surface in the same curved direction. Figures 6(i)–(l) depict the intensity distributions of the topological localized states for the different cases.

5. Discussion

We developed a 2D curved TI by perforating a curved rigid surface, and demonstrated that localization of the topological edge states with an increasing geometrical curvature. Our experiments directly revealed the localized state at the top of the curved structure, and agreed very well with the simulation. Moreover, we demonstrated that the localized edge state is robust against defect disturbance. To understand the physical

mechanism, we theoretically derived the geometric potential distribution on the curved surface based on conformal transformation, and implemented the tight-binding model with modified onsite energy by considering the discrete geometric potentials. The theoretical results explained the origin of the localized state, in which the geometric potentials modulated the onsite energy of the holes on the curved surface to form a bilaterally symmetric distribution. The novel approach for manipulating the topological edge states from the propagated mode to the localized mode controlled by the geometric curvature have potential practical applications in flexible devices.

6. Method

6.1. Simulations

Numerical simulations in this work are all performed using the 3D acoustic module of a commercial finite-element simulation software (COMSOL Multiphysics). The resin blocks are treated as acoustically rigid materials. The mass density and sound velocity in air are taken as 1.21 kg m^{-3} and 346 m s^{-1} , respectively. In the eigen evaluations, four boundaries of the unit cells are set as Floquet periodic boundaries for the data in figures 1(b) and (c). To suppress the edge states and corner states along the outside boundaries of the structure, the boundaries of the whole structure are set as plane wave radiation boundaries except the rigid bottom, for the data in figures 2, 6 and S3.

6.2. Experiments

In order to verify our design, the proposed structure was precision-fabricated using epoxy resin via 3D printing, where the mass density and sound velocity in air are taken as 1050 kg m^{-3} and 2400 m s^{-1} , respectively. The fabricated sample in figure 3(a) consisted of 768 holes with the depth 3 cm and radius 0.3 cm. The sample was set up in the free space which is surrounded by the absorbing sponges. A point-like sound source was designed by connecting one loudspeaker to the metallic pipes, which was placed near the structure of the holes at a distance of 3 cm. The structural local pressure fields were measured by inserting the microphone into the bottom of hole. The bottom will be blocked after measure to keep the existence of spoof acoustic surface wave supported in the structure. When measuring the pressure spectrum with frequency as the variable, the measurement interval is 10 Hz. The signal generator (AFG1022, Tektronix) amplified by a power amplifier (CPA2400, SinoCinotech), then the digitizer acquired the outputs of the microphones (378B02, PCB Piezotronics). The transmission spectra of the bulk, edge, and localized state were obtained by measuring transmission pressure fields. The indoor temperature was strictly controlled within $25 \pm 0.3 \text{ }^{\circ}\text{C}$ during the whole experiment. Ambient noise is lower than 20 dB, the influence on the experiment can be negligible.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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H W W conceived the idea and did the theoretical analyses. J Q Q performed the numerical simulations. J Q Q and Y K L performed experimental measurements. H W W, Z Q S guided the research. All authors contributed to discussions of the results and paper preparation. H W W and J Q Q wrote the paper.

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