

Observation of Photonic Topological Floquet Time Crystals

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The two main research questions on time crystals are: What is a time crystal? Is there a material that is spontaneously crystalline in time? This study synthesizes a photonic material of topological Floquet time crystals and experimentally observes its indicative period- $2T$ beating. A single-particle picture is explicitly reconstructed of discrete time-crystalline phase and is revealed using an appropriately-designed photonic Floquet simulator the rigid period-doubling as a signature of the breakage of the discrete time-translational symmetry. Unlike the requirement of the many-body localization, the photonic Floquet time crystal is derived from a newly defined single-particle topological phase that can be extensively accessed by many pertinent nonequilibrium and periodically-driven platforms. The observation will drive theoretical and technological interests toward condensed matter physics and topological photonics.

1. Introduction

A time crystal is an exotic nonequilibrium state of matter that repeats itself in space and time and spontaneously breaks the continuous or discrete time-translational symmetry. It mimics an ordinary crystal's ability to spontaneously break its spatial translation symmetry in space dimension when cooling down. In 2012, Wilczek first proposed the concept of time crystals in both the classical and quantum variants;^[1,2] however, in the subsequent

years, the original time-crystalline model was proved to be invalid in static equilibrium and later ruled out by a no-go theorem.^[3,4] A recent work^[5] indicates the ground-state existence of continuous time crystal in the presence of multi-body long-range interactions, even for time-independent systems. On the other hand, breakage of the discrete time-translational symmetry in periodically driven systems has not been ruled out,^[6–9] and it becomes a promising research direction.^[10–15] To date, most materialized time crystals follow this direction and are rapidly explored in a number of quantum simulation platforms such as trapped ions,^[16] diamond nitrogen-vacancy centers,^[17] superfluid quantum gases,^[18,19]

and nuclear magnetic resonances.^[20,21] However, the ground state (a state of minimum energy) of a genuine time-crystalline phase is elusive^[5] because it is intrinsically non-conservative out-of-equilibrium after its time-translational symmetry is spontaneously broken due to the temporal periodicity.

Usually, the existence of a discrete time-crystalline phase can be assessed by probing a stable subharmonic response in experiments, such as period doubling (with twice the period of the underlying drive). However, time crystals are not the only materials or systems that can give rise to subharmonic period- $2T$ oscillation.^[16,22] Indeed, even before the inception of the discrete time-crystalline phase, period- $2T$ oscillations have been widely studied in a variety of classical and quantum systems, such as period-doubling bifurcation from a logistic map,^[23] subharmonic response in chaotic^[24–26] or dissipative^[10,27,28] systems, and coupled nonlinear parametric oscillators (e.g., van der Pol oscillator).^[22,26] For example, in a period-doubled dissipative system, energy is supplied from the drive to the system and then is reciprocally released by the dissipation. As a result, the balance between the driving force and dissipation can avoid the infinite thermalization of such a system and yield to a dissipative time crystal. Moreover, several fascinating candidates for time-crystalline systems have been proposed recently.^[5,18,29–33] In other words, the crucial subharmonic hard evidence required to experimentally substantiate a time-crystalline phase is disputed.

To address the debate, the following question must be posed: Does the time-crystalline phase of matter possess many-body collective phenomenon, a precursor to chaotic dynamics, or single-particle subharmonic excitation? No singular answer to this question has been proposed. To explore the genuine phenomenon of time crystals, more experiments are needed. Hence, this paper explores the construction of a single-particle picture of time

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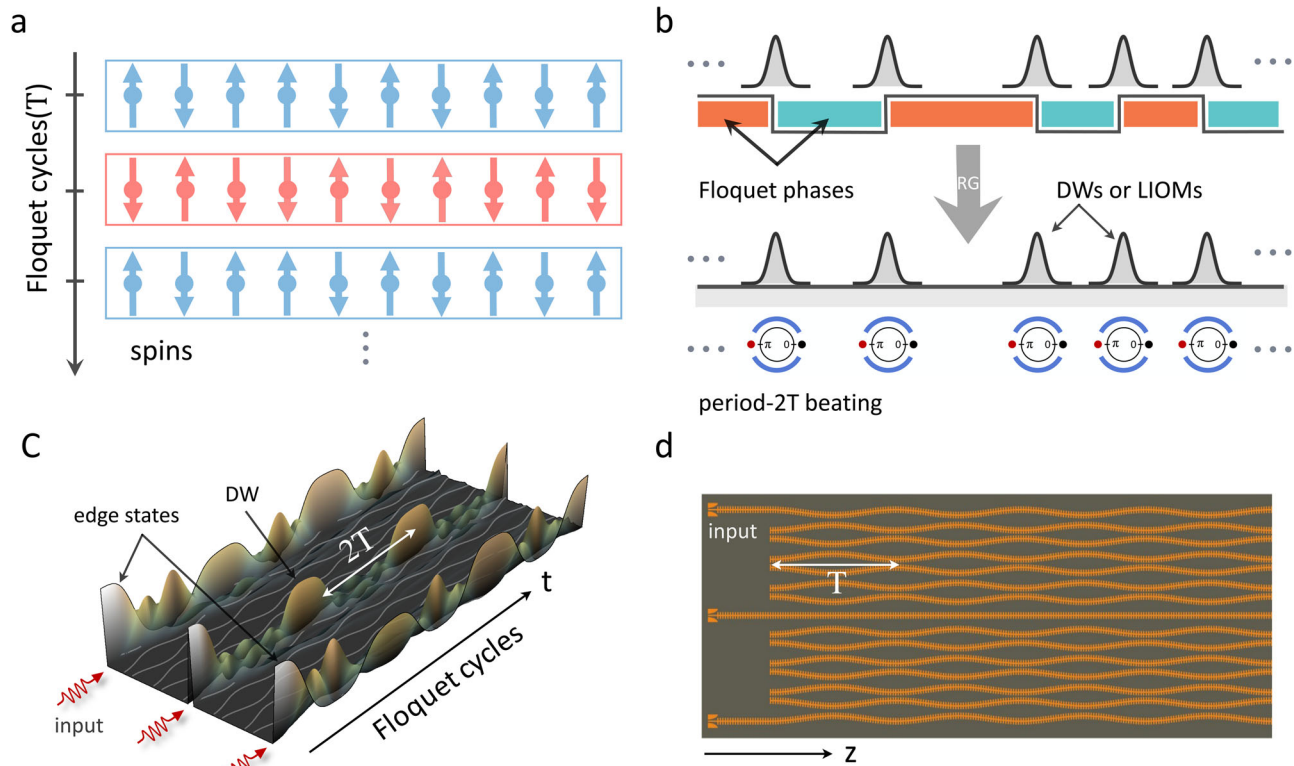


Figure 1. Period-2T Floquet time-crystalline phases in topologically protected single-particle pictures. a) Many-body-localization-enabled Floquet time crystal on a spin chain. b) Emergence of domain walls (DWs) or local integrals of motions (LIOMs) in 1D disordered Floquet phases. The superposition of Floquet topological invariants zero and π modes of the renormalized low-lying DW excitations offers the rigid period-doubling beating as a time-crystalline clock. c) Schematic of the stroboscopic evolutions of DWs and edge states on a chain (e.g., spins, atoms, and waveguides) with a periodically driven protocol, demonstrating the pertinent period-2T beating. d) Schematic of the curved waveguide array structure with three field inputs.

crystals that enables a rigid period-2T oscillation.^[14,34,35] Inspired by the discoveries of topological insulators^[36,37] and topological photonics,^[38] we attempt to experimentally realize a robust photonic Floquet time crystal in a platform of waveguide arrays that can facilitate the development of periodically driven time-crystalline topological phases. To this end, our observation indicates a universal presence of Floquet time-crystalline states in topological materials and systems.^[14,15,34,35,39,40]

2. Results

Let us first formulate an analytical expression to elucidate the single-particle picture of a Floquet or discrete time crystal (FTC, or DTC). To illustrate the spin chain, as schematically shown in **Figure 1a**, a many-body-localization-enabled time-crystalline state (i.e., π -spin glass) was intensively studied,^[7–9,41] given by $|\{\pm\}\rangle = (|\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \dots\rangle \pm |\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \dots\rangle) / \sqrt{2}$ (also called Schrödinger's cat states). Notably, the π -spin glass states $|\{\pm\}\rangle$ are the eigenstates of the Floquet evolution operator $U_F |\{\pm\}\rangle = e^{-i\epsilon_{\pm} T} |\{\pm\}\rangle$; consequently, period-doubling was expected due to energy splitting, i.e., $|\epsilon_+ - \epsilon_-| = \pi/T$, during a driving period T . This fully entangled many-body ground state is dubbed “ π SG/DTC” (π -spin glass/discrete time crystal).^[9] By emulating

the π SG/DTC Floquet states, a topologically protected Floquet product state can be conjectured, suggested as

$$|\Psi_{FTC}\rangle = |\text{edge state}\rangle \otimes |DW\rangle \otimes |DW\rangle \otimes \dots \otimes |\text{edge state}\rangle \quad (1)$$

where the edge state ($\sim \frac{|\uparrow\rangle \pm |\downarrow\rangle}{\sqrt{2}}$) is analogous to the disorder-protected quantum order at the ends, and the domain walls (DWs) ($\sim \frac{|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle}{\sqrt{2}}$) are related to the protected local integral of motions (LIOMs) randomly distributed on the chain.^[41,42] The conjecture stems from the equivalence between the LIOMs and topological domain walls.^[42–45] Thus, the constructed many-body time-crystalline state (Equation (1)) consists of the direct product of these local single-particle topological edge states and domain walls, as shown in **Figure 1b**. In other words, the MBL spin glass can be characterized by localized nonoverlapping domain walls, while the thermal states by extended domain walls.^[42,44] Correspondingly, as shown in **Figure 1b**, the domain walls are pinned at the border between different Floquet topological phases. As a result, MBL-enabled π SG/DTC phase can be phenomenologically mapped into a bunch of domain walls.^[45,46] Still, we should note that even though the product state (Equation (1)) can display period doubling, it is radically different from $|\{\pm\}\rangle$, because there is no entanglement between domain walls due to the absence of many-body interaction. The mapping from transverse field Ising

model to a Majorana-type SSH model can be founded in the Supporting Information.

Consider a topological Floquet phase holding both zero- and π -quasienergy modes at the ends or in the kink structures.^[34,47] Correspondingly, the edge state and DW excitations can be described as the superposition states of two topological modes, i.e.,

$$|sdge\rangle = \frac{|0\rangle \pm |\pi\rangle}{\sqrt{2}}, |DW\rangle = \frac{|0DW\rangle \pm |\pi DW\rangle}{\sqrt{2}} \quad (2)$$

As shown in Figure 1b, a period- $2T$ beating occurs due to the superposition of zero ($|0\rangle, |0DW\rangle$) and π quasienergy eigenstates ($|\pi\rangle, |\pi DW\rangle$). The dynamic intensity of a local superposition Floquet state is prototypically given by $I_{\pm}(x, t) = |\psi_0(x, t) \pm \psi_{\pi}(x, t)|^2 = |\psi_0(x, t)|^2 + |\psi_{\pi}(x, t)|^2 \pm 2\Re\{\psi_0^*(x, t)\psi_{\pi}(x, t)\}$. The wavefunctions $\psi_{0,\pi}(x, t)$ represent the generic 0 and π Floquet states being projected into the discrete lattice (x) coordinates, that is $\psi_0(x, t) = \langle x|0\rangle$, $\psi_{\pi}(x, t) = \langle x|\pi\rangle$. According to the Floquet–Bloch theorem,^[42,47] the two Floquet eigenstates are given by $\psi_{0,\pi}(x, t) = u_{0,\pi}(x, t)e^{-i\epsilon_{0,\pi}t}$, where $u_{0,\pi}(x, t) = u_{0,\pi}(x, t+T)$ defines the micromotion within one cycle and $\epsilon_{0,\pi}$ are the corresponding quasienergies. Therefore, the intensity of superposition is obtained as follows:

$$I_{\pm}(x, t) = |u_0(x, t)|^2 + |u_{\pi}(x, t)|^2 \pm 2\Re\{u_0^*(x, t)u_{\pi}(x, t)\} \times \cos(|\epsilon_0 - \epsilon_{\pi}|t) \quad (3)$$

Period-doubling occurs if the quasienergy difference $|\epsilon_0 - \epsilon_{\pi}| = \pi/T$ is locked. We can easily corroborate the relation $I_{\pm}(x, t+2T) = I_{\pm}(x, t)$, so that the intensity evolves with a double period of the drive, as shown in Figure 1c. Namely, the $2T$ -periodic subharmonic response stems from the interference between the symmetry-protected Floquet eigenmodes. Two facets are worth mentioning here. First, it is not difficult to pick two quasienergy states with the splitting π/T in the Floquet–Brillouin zone ($-\frac{\pi}{T}, \frac{\pi}{T}$) and realize the period- $2T$ beating. However, the problem is that this type of subharmonic response is fragile as it can be destroyed by an infinitely small perturbation. The vulnerability of this period-doubling reflects that the splitting can be quickly affected by other Floquet eigenstates, and period- $2T$ interference (the third term in Equation (3)) is perturbed. Given this issue of stability, only the nontrivially gapped Floquet modes are immune against scattering from imperfections and disorders.^[34]

Second, the relevant subharmonic interference must be local, i.e., $2\Re\{u_0^*(x, t)u_{\pi}(x, t)\} \neq 0$, and consequently the 0 or π edge states and DWs are located at the same ends or kinks. The micromotions of the different spatially located 0 and π modes are too far apart to overlap, yielding no interference. Thus, only the local superposition between 0 and π modes predominantly contributes to the emergent period- $2T$ oscillation.

2.1. Setup

The photonic FTC was modeled after the Su–Schrieffer–Heeger (SSH) model for polyacetylene (1979),^[50] which has been widely investigated on many photonic simulation platforms (e.g.,^[38]).

The edge states and DWs can be modeled in a driven SSH model using waveguide arrays (Figure 1d). For easy implementation in photonic systems, we chose the periodically driven SSH chain. The Hamiltonian of this biatomic model is given by $H(t) = \sum_{i=1}^{N-1} [\kappa_0 + (-1)^i(\delta\kappa_0 + \delta\kappa(t))]c_i^\dagger c_{i+1} + h.c.$, where $c_i^\dagger(c_i)$ are the creation (annihilation) operators of the light field amplitude on the i^{th} waveguide.

The time-periodic coupling term between two nearest-neighboring waveguides (or sites) was dimerized, $\kappa_{i,i+1}(t) = \kappa_0 + (-1)^i(\delta\kappa_0 + \delta\kappa(t))$, where κ_0 is the constant coupling strength, and $\delta\kappa_0$ and $\delta\kappa(t)$ are the time-independent staggered coupling strengths due to global dimerization and time-periodic dimerization, respectively. $\delta\kappa(t) = \delta\kappa_1 \cos(\omega t + \theta)$, where $\delta\kappa_1$ is the strength of the coupling, $\omega = 2\pi/T$ is the Floquet driven frequency, and θ is the initial phase (Floquet gauge) of the drive. Equivalently, the topological phases in the driven SSH model can be mapped onto the transverse field Ising model^[7] or the Kitaev model for a p -wave superconductor.^[14,35]

In the experiment, the periodic coupling $\kappa_{i,i+1}(t)$ was appropriately designed and fully controlled by the spatial spacing (G) between two neighboring curved waveguides. As shown in Figure 1d, a photonic simulator of driven SSH chain was designed by mapping the evolution time t of an electron in the direction of light propagation z , and correspondingly mapping the Floquet cycle T onto the curving period Λ . Thus, the coupling profile was given by $\kappa_{i,i+1} = \kappa_{i,i+1}(G, \Lambda)$. For demonstration, we defined an effective coupling length $l_c = \pi/2\kappa_0$ of the simulator to compare it with the period Λ . The dimerization conditions required for further fabrication were $\delta\kappa_0 \ll \kappa_0$ and $\delta\kappa_1 \ll \kappa_0$. Typically, the coupling length l_c is in the range of 20–100 mm, and the curving period is fixed to $\Lambda = 100$ mm. The coupling profiles extracted from the simulations can be found in the Supporting Information file.

2.2. Quasienergy Gap Opening

Figure 2 demonstrates the quasienergy band for the emergent topological phase coexistence of both zero and π Floquet modes. The quasienergy spectrum was precisely calculated using the eigenvalue problem analyses of the Floquet Hamiltonian

$H_F = \frac{i}{\Lambda} \log \mathcal{T} \exp(-i \int_0^\Lambda H(z) dz)$,^[47–49] in which the Hamiltonian is time-periodic $H(z) = H(z + \Lambda)$. Figure 2a presents the desired band as a function of the Floquet cycle (Λ) with respect to the effective coupling length (l_c). Given the two dimerizations ($\delta\kappa_0 \neq 0$, $\delta\kappa_1 \neq 0$), the two Floquet modes coexist at the periodically curving condition $\Lambda/l_c \in (1, 2)$, which is associated with two quasienergy gap invariants^[47] (see the Supporting Information file).

Correspondingly, in the region of coexistence, eigenstates of the 0 and π modes are illustrated in Figure 2b. We plotted their evolutionary patterns over four cycles ($4T$), showing that both Floquet modes are periodic in T . The intensity of the 0 mode is mainly localized on the first waveguide of the boundary of the array. The π -mode periodically propagates along with the first two waveguides. Notably, the sole eigenstate excitation of either 0 mode or π mode cannot produce the period-doubling oscillation.

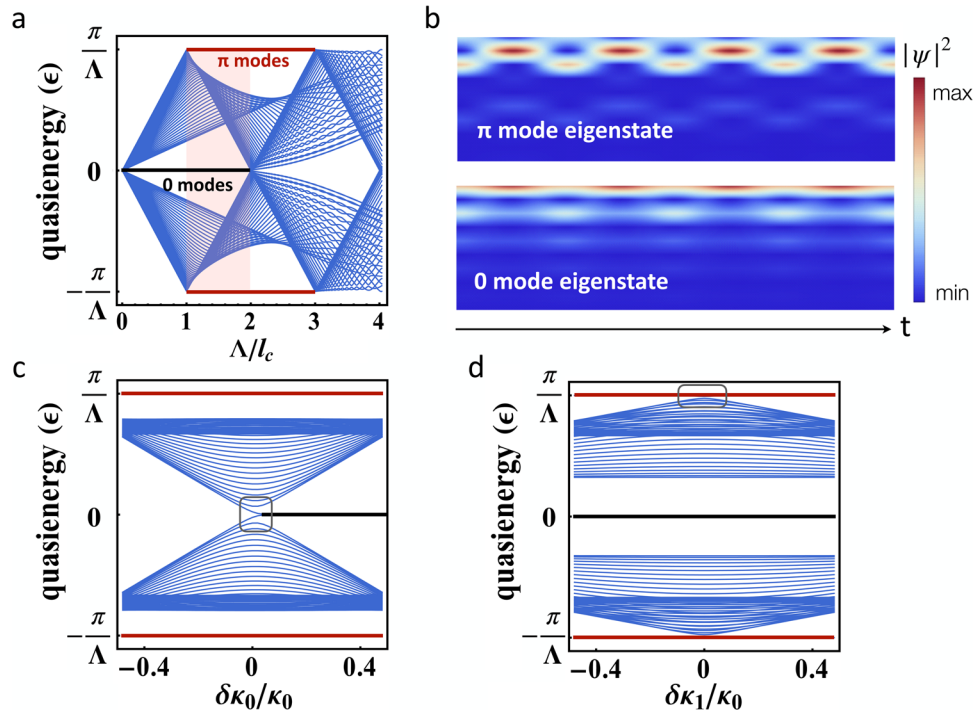


Figure 2. Quasienergy spectrum of photonic FTC and micromotion eigenstates of topological 0 and π modes. a) Quasienergy band as a function of the curving period (Λ). b) Micromotions of π -mode and 0-mode eigenstates in four Floquet cycles ($4T$). Floquet cycle T is mapped to the curving period Λ . The two Floquet topological modes coexist in the driven condition $\Lambda/l_c \in (1, 2)$, and the coupling length $l_c = \pi/2\kappa_0$. The parameters are $\kappa_0 = 0.25$, $\delta\kappa_0 = 0.06$, and $\delta\kappa_1 = 0.12$. c) Zero-gap as a function of global dimerization $\delta\kappa_0/\kappa_0$, closed at $\delta\kappa_0/\kappa_0 = 0$. d) π -gap as a function of time-period dimerization $\delta\kappa_1/\kappa_0$, closed at $\delta\kappa_1/\kappa_0 = 0$. The setting for both (c) and (d) is the Floquet frequency $\Lambda/l_c = 4/3$.

Figure 2c,d demonstrates the dependence of zero- and π -gap opening on the staggered strengths $\delta\kappa_0$ and $\delta\kappa_1$, respectively. In a fixed cycle in the Floquet coexistence region, the zero- and π -gaps closed at $\delta\kappa_0 = 0$ and $\delta\kappa_1 = 0$, separately, as depicted in the insets, and then opened linearly as the dimerization strengths increased. The difference between the two modes is that, while the 0 mode would disappear at $\delta\kappa_0 < 0$ (corresponding to a trivial phase), the π mode still exists at $\delta\kappa_1 < 0$. The reason is that for the negative periodic staggered coupling strength, $\delta\kappa_1 \cos(\omega t + \theta) = \delta\kappa_1 \cos(\omega t + \theta + \pi)$ corresponds to the π -phase shift for a Floquet gauge choice. That is, the π modes emerging with the negative dimerization coupling ($-\delta\kappa_1$) and gauge (θ) are equivalent to the modes with positive dimerization coupling ($+\delta\kappa_1$) and gauge ($\theta + \pi$). Therefore, the advantage of this array design is that the global dimerization and time-period dimerization solely control the opening of the 0- and π -gaps, respectively. This dependence made it convenient to control and demonstrate our theoretical expectations using the photonic simulation and measurement.

2.3. Period-2T Beating in Floquet Simulator

The photonic simulator was made up of coupled ultrathin corrugated copper strips that support spoof surface plasmon polaritons (SPPs) propagating at microwave frequencies as the highly confined guided wave on a plasmonic waveguide. These ultrathin microstrip lines were first proposed by^[51] and were recently implemented as a simulation platform to observe anomalous topo-

logical modes.^[52] These ultrathin waveguides were deposited on a flexible dielectric substrate (F4BK) that can be bent, folded, coiled, and twisted to guide the spoof SPPs.^[51] Prior to the array fabrication, we used a finite element method in a commercial software (COMSOL Multiphysics) to numerically simulate the near-field distribution of the TM-polarized wave (perpendicular to the ultrathin metallic waveguide interface) propagating along the z-direction on the proposed array. For details regarding the fabrication, measurement, and simulation (see the Supporting Information file).

Figure 3 compares the theoretical expectation, numerical simulation, and experimental observation of the time-crystalline stroboscopic evolution of topological superposition. As observed from the experiment in Figure 3c, the period- $2T$ oscillation in the curved array sample was probed, and the results perfectly agreed with the prediction (Figure 3a) and simulation (Figure 3b). The array consists of ten curved waveguides ($N = 10$) with a period length (Floquet cycle) $\Lambda = 100$ mm and a total length $L = 400$ mm. Its structural fabrication mimics the periodically driven SSH model by considering both global and time-periodic dimerizations. The experimental near-field intensity pattern was scanned by a metallic tip to detect the electric field (E_z) over the simulator surface, which was connected to a network analyzer to collect data and perform near-field distribution imaging (see the Supporting Information file).

With the coupling profiles corresponding to the structural parameters, we calculated the dynamic evolution from the first waveguide of the array. Figure 3a illustrates the intensity distribu-

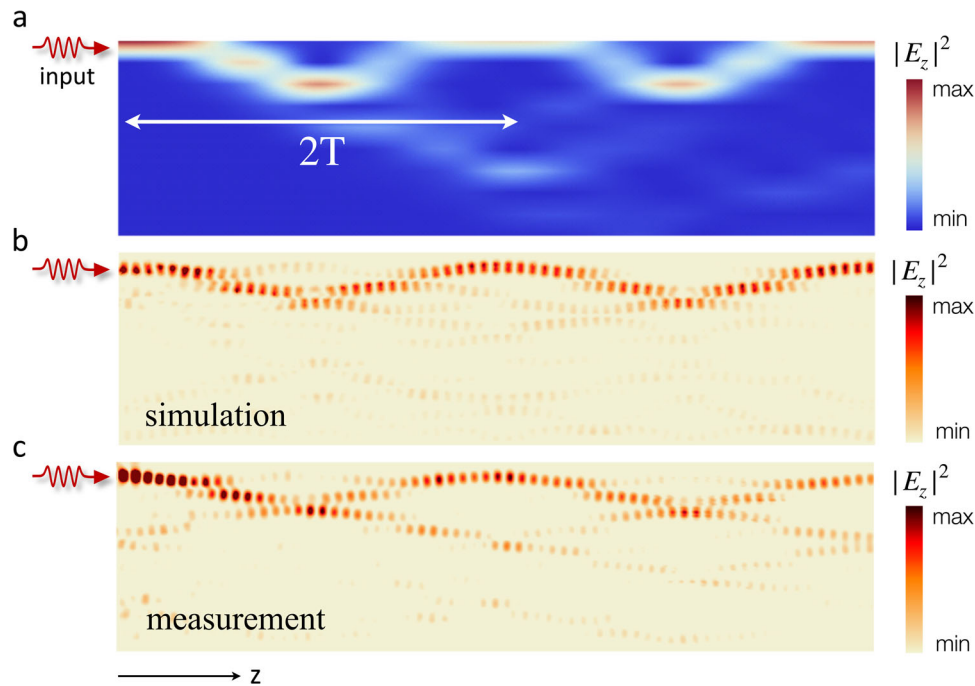


Figure 3. Observation of period- $2T$ oscillation in photonic Floquet simulator. a) Time evolution of the driven SSH model based on the Floquet evolution operator. b) Finite-element method (FEM) simulation based on the commercial software COMSOL multiphysics. c) Near-field measurement based on a fabricated array sample. The field was input from the edge of the waveguide array (waveguide number $N = 10$) with an input frequency of 17.0 GHz for the simulations and 16.9 GHz for the experiments. The length of the simulator is $L = 400$ mm. The theory, simulation, and experiment are in perfect agreement with each other.

tion within four Floquet cycles ($4T$), demonstrating the period- $2T$ oscillation twice along the array's boundaries. As observed in the quasienergy band, the input field was mainly projected on to the chiral-symmetry-protected edge modes ($|\psi_{in}\rangle = \alpha|0\rangle + \beta|\pi\rangle$). After N Floquet cycles, the stroboscopic dynamics of the superposition state were given by $U_F(NT)|\psi_{in}\rangle = \alpha|0\rangle + (-1)^N\beta|\pi\rangle$. Thus, the resulting period-doubling appears because the period number (N) must be even for the renewal of its initial state ($|\psi_{in}\rangle$).

3. Discussions

3.1. Topological Floquet Phase Transition

A close look at the quasienergy band outside the coexistence region (Figure 2a) reveals that only the zero modes survive in the high-frequency-driven region $\Lambda/l_c \in (0, 1)$, and only the π -mode appears in the intermediate region $\Lambda/l_c \in (2, 3)$. By decreasing the curving period, sequent harmonic (period- T), subharmonic (period- $2T$), and static responses can be expected on the boundaries because the corresponding Floquet systems undergo a phase transition from the π mode, topological phase coexistence, and zero modes.^[34] For sample fabrication, a simpler way to detect this transition is by altering the ratio between the global and periodic dimerizations.

Figure 4 demonstrates the experimental near-field observation of the topological transition from a harmonic (T) and subharmonic ($2T$) to a static response. Figure 4a illustrates the π -mode excitation, compared with the case $\delta\kappa_0 = 0$ in Figure 2c. Then,

we increased the global staggered distance between two neighboring waveguides. Zero-modes emerged owing to the nontrivial zero-gap opening via increasing global dimerization. Likewise, Figure 4b illustrates the subharmonic periodic- $2T$ evolution owing to the coexistence of the two modes. Finally, at the almost fully dimerized limit (corresponding to no coupling between the first two waveguides), only the first waveguide can propagate through the input field, indicating the existence of an extremely isolated zero mode, but the π mode is suppressed. As shown in Figure 4c, most of the input fields remain on the first waveguide, demonstrating the zero-mode isolation. However, few input fields diffuse into the array owing to the effective surplus residual coupling between waveguides. In addition, we thoroughly investigated the dependence of Floquet cycles and input frequencies in the Supporting Information file.

3.2. Subharmonic Oscillation in Both Edge States and Domain Walls

The time-crystalline subharmonic response appears on the boundaries of a chain and can also emerge in the bulk as DWs (as the interfacial modes between Floquet phases). As depicted in Figure 1d, the two Floquet edge states lie at the opposite ends, and a single DW is positioned in the middle of the array. The existence of DWs depends only on the global topological difference of both sides,^[34] while it is free from the local coupling profiles. The principle of bulk-edge correspondence supports DW excitations.^[36] To retain the structural symmetry of the photonic

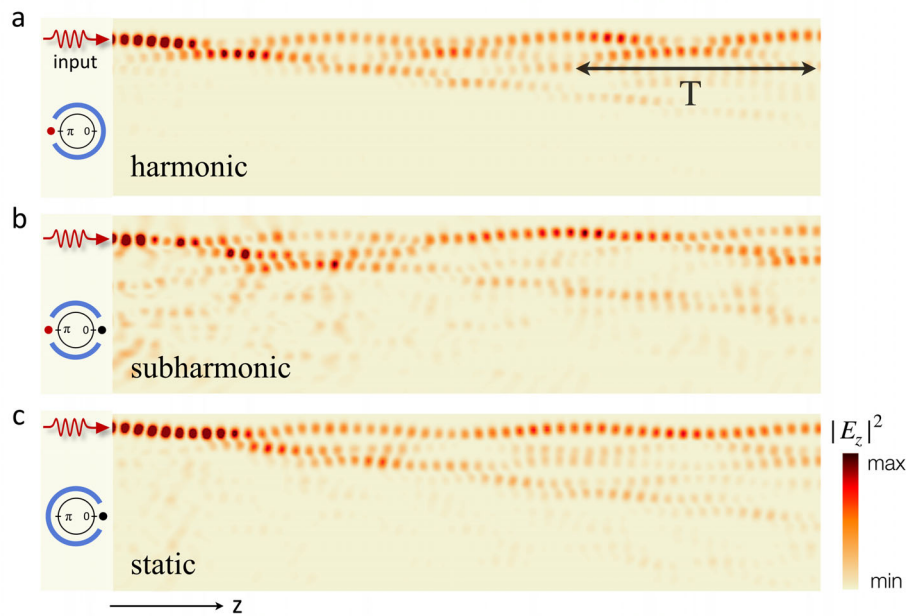


Figure 4. Observation of topological phase transition in Floquet simulators. By altering the ratio between the periodic and global dimerization spacing, we can open the corresponding quasienergy gap, as shown in the insets. a) For $G_{\min 1} = 0.9$ mm and $G_{\min 2} = 0.9$ mm, only π mode exists, and the photonic system exhibits the period- T oscillation when the π mode is excited. b) For $G_{\min 1} = 1.9$ mm, $G_{\min 2} = 0.9$ mm, 0 and π modes coexist, and the system exhibits period- $2T$ oscillation when the two modes are excited simultaneously. c) For $G_{\min 1} = 2.3$ mm and $G_{\min 2} = 0.9$ mm, only 0 mode is effectively excited. The system exhibits nondriven static behavior. Note that the input frequency is 17.4 GHz, array length $L = 300$ mm, and the Floquet cycle $3T$.

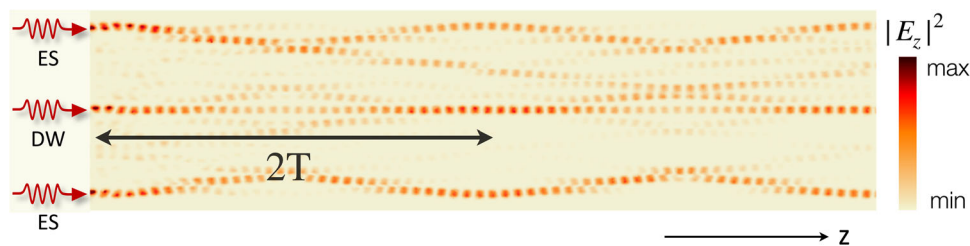


Figure 5. Demonstration of topological discrete time crystals with inputs from both edge states (ES) and a central DW. Near-field simulation with combined edge states and domain walls demonstrates the period- $2T$ oscillation. The structural parameters are $N = 15$, $L = 400$ mm, and $\Lambda = 100$ mm, and the input frequency is 17.0 GHz.

simulator, the middle waveguide was set as straight (Figure 1d). Nevertheless, the local central couplings to the laterally curved waveguides were still periodically modulated. We conclude that both the end states and DWs permit the coexistence of the two anomalous Floquet modes in this design.

Figures 5 shows the simulated near-field distributions of the subharmonic time-crystalline states with three combined inputs of a central DW and two edge states (ES) at ends. The input frequency was 17.0 GHz and divided into four cycles, while the array length was the same 400 mm. The waveguide number was increased up to $N = 15$ with a central straight waveguide on the 8th site to hold the DW. In this simulation, the field distribution propagated along the waveguides oscillating with a period twice that of the curving. The local period-doubling behavior was rigid, owing to the spatial separation of the topological excitations, so that it does not suffer finite-size hybridization.

We notice that the domain walls and edge states in generic topological systems are inevitably suffering from the finite size effect, and for the SSH setting, their topological protection of rigid π/T -splitting is also vulnerable to the local on-site disorders which break the sublattice symmetry.^[34] Interestingly, the overlapping between domain walls can be applied to demonstrate the MBL-to-thermalization phase transition.^[42] These time-crystalline domain walls and edge states behave as a protected “boundary time crystal”, which emerges from the borders between distinct topological Floquet phases (see Figure 1b), instead of requiring macroscopic dissipation of open systems in the thermodynamic limit.^[11]

Also, it is worth noting that to mimic a genuine many-body time crystal is a challenge for our Floquet simulator based on the microwave waveguides. For photonic realization, mimicking the many-body entanglement between domain walls re-

quires the waveguides to be optically nonlinear^[53] and be excited by quantum light inputs (e.g., the entangled photons). This requirement exceeds the capability of the proposed Floquet simulator.^[51] Therefore, we leave this challenge to future research on quantum many-body platforms^[16–21] and all-optical nonlinear microcavities.^[54]

In brief, we designed a photonic-material alternative of Floquet time crystal and observed its prototypical period-doubling behavior in our Floquet simulators. To avoid many-body interaction in the photonic simulation, we reconstructed a topologically protected time-crystalline state composed of Floquet topological edge states and domain walls. In a first, both single-particle and many-body pictures of discrete time-crystalline phases were demonstrated. We believe that the noninteracting topological Floquet time crystals can be easily extended for implementation on many classical and quantum simulation platforms. Also, we hope that the photonic Floquet time crystal can shed light on exotic time-crystalline phase transitions and spur the further development of the out-of-equilibrium state of matter in photonics and condensed matter.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Keywords

Floquet topological photonics, quantum simulation in waveguide arrays, time crystals, topologically protected period doubling

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